

## X-bar and R Control Charts



An **X-Bar and R-Chart** is a type of statistical process control chart for use with continuous data collected in subgroups at set time intervals - usually between 3 to 5 pieces per subgroup. The Mean (X-Bar) of each subgroup is charted on the top graph and the Range (R) of the subgroup is charted on the bottom graph. Out of Control points or patterns can occur on either the X-bar or R chart.

Like all control charts, an X-Bar and R-Chart is used to answer the following questions:

- Is the process stable over time?
- What is the effect of a process change on the output characteristics?
- How will I know if the process becomes unstable, or the performance changes over time?

### When is it used?

- Constructed throughout the DMAIC process, particularly in the Measure, Analyze and Control phases of the cycle.
- Used to understand process behavior, evaluate different treatments or methods, and to control a process.
- Recommended for **subgroup sizes of 10 or less**. If the subgroup size exceeds 10, the range chart is replaced by a chart of the subgroup standard deviation, or S chart.

**NOTE:** It has been estimated that 98% of all processes can be effectively represented by using either the XmR charts or X-Bar & R charts.

(Donald Wheeler)

### How to Construct an X-Bar and R Control Chart

X-Bar and Range (or S) charts are always constructed and viewed as a pair.

To construct an X-Bar and R Chart, follow the process steps below. For subgroup sizes greater than 10, substitute the subgroup standard deviation (S) for range (R), and use constants for S from the table located after the instructional steps.

1. **Record** subgroup observations.
2. **Calculate the average (X-Bar) and range (R)** for each subgroup.

3. **Plot the X-bar and R values** for each subgroup in time series. You can create a meaningful control chart from as few as 6-7 data points, although a larger sample size (20+ subgroups) will provide much more reliability. In most cases, control limits are not calculated until at least 20 subgroups of data are collected.
4. **Calculate the average R value**, or R-bar, and plot this value as the centerline on the R chart.
5. Based on the subgroup size, select the appropriate constant, called D<sub>4</sub>, and multiply by R-bar to **determine the Upper Control Limit** for the Range Chart. All constants are available from the reference table.

$$UCL(R) = R\text{-bar} \times D_4$$

Plot the Upper Control Limit on the R chart.

6. If the subgroup size is between 7 and 10, select the appropriate constant, called D<sub>3</sub>, and multiply by R-bar to **determine the Lower Control Limit** for the Range Chart. There is no Lower Control Limit for the Range Chart if the subgroup size is 6 or less.

$$LCL(R) = R\text{-bar} \times D_3$$

Plot the Lower Control Limit on the R chart.

7. Using the X-bar values for each subgroup, **compute the average of all X-bars**, or X-bar-bar (also called the Grand Average). Plot the X-bar-bar value as the centerline on the X Chart.
8. **Calculate the X-bar Chart Upper Control Limit**, or upper natural process limit, by multiplying R-bar by the appropriate A<sub>2</sub> factor (based on subgroup size) and adding that value to the average (X-bar-bar).

$$UCL(X\text{-bar}) = X\text{-bar-bar} + (A_2 \times R\text{-bar})$$

Plot the Upper Control Limit on the X-bar chart.

9. **Calculate the X-bar Chart Lower Control Limit**, or lower natural process limit, for the X-bar chart by multiplying R-bar by the appropriate A<sub>2</sub> factor (based on subgroup size) and subtracting that value from the average (X-bar-bar).

$$LCL(X\text{-bar}) = X\text{-bar-bar} - (A_2 \times R\text{-bar})$$

Plot the Lower Control Limit on the X-bar chart.

10. After constructing the control chart, follow the same rules to **assess stability** that are used on XmR charts. Make sure to evaluate the stability of the Range

Chart before drawing any conclusions about the Averages (X-Bar) Chart --- if the Range Chart is out of control, the control limits on the Averages Chart will be unreliable.

### Control Chart Constants

	X-bar and R Chart			
Subgroup Size (n)	$A_2$	$D_3$	$D_4$	$d_2$
2	1.880	0	3.267	1.128
3	1.023	0	2.574	1.693
4	0.729	0	2.282	2.059
5	0.577	0	2.114	2.326
6	0.483	0	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078

	X-bar and S Chart		
Subgroup Size (n)	$A_2$	$B_3$	$B_4$
11	0.927	0.322	1.678
12	0.886	0.354	1.646
13	0.850	0.382	1.619
14	0.817	0.407	1.593
15	0.789	0.428	1.572



**Exercise:** Following is a table of data sampled twelve times from a process; the process mean is supposed to be 74.000 inches. Plot these data in an X-bar R chart to determine if the process is in statistical control (note: n = 6).

Sample #	Observations						Average	Range	
1	74.030	74.002	74.019	73.992	74.006	73.999	74.008	0.038	
2	73.989	74.000	74.001	73.995	73.998	74.000	73.997	0.012	
3	73.998	74.002	74.050	73.989	74.005	74.030	74.012	0.061	
4	74.005	74.030	73.995	74.000	73.998	73.990	74.003	0.040	
5	74.006	73.990	74.004	73.998	74.000	74.002	74.000	0.016	
6	73.982	73.984	73.998	73.999	73.998	73.995	73.993	0.017	
7	74.003	74.002	74.008	73.992	74.006	73.999	74.002	0.016	
8	73.989	74.001	74.001	73.998	73.998	74.000	73.998	0.012	
9	73.998	74.002	74.001	73.989	74.005	74.030	74.004	0.041	
10	74.005	74.021	73.995	73.990	73.998	73.993	74.000	0.031	
11	74.004	73.995	74.004	73.998	74.000	74.002	74.001	0.009	
12	73.982	73.984	73.998	73.999	73.998	73.995	73.993	0.017	
x-double-bar							74.001	R-bar	0.026